Exclusive charmless B_s hadronic decays into η' and η

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Using the next-to-leading order QCD-corrected effective Hamiltonian, charmless exclusive nonleptonic decays of the B_s meson into η or η' are calculated within the generalized factorization approach. Nonfactorizable contributions are included with two different treatments. Some subtleties involved are discussed.

1 Introduction

Stimulated by the recent observations of the large inclusive and exclusive rare B decays by the CLEO Collaboration [1], there are considerable interests in the charmless B meson decays [2]. To explain the abnormally large branching ratio of the semi-inclusive process $B \to \eta' + X$, several mechanisms have been advocated [3,4,5,6] and some tests of these mechanisms have been proposed [7]. It is now generally believed that the QCD anomaly [3,4,5] plays a vital role. The understanding of the exclusive $B \to \eta' K$, however, relies on several subtle points. First, the QCD anomaly does occur through the equation of motion [8,9] when calculating the $(S-P)(S+P)$ penguin operator and its effect is found to reduce the branching ratio. Second, the mechanism of $c\bar{c} \to \eta'$, although proposed to be large and positive originally [10,11], is now preferred to be negative and smaller than before as implied by a recent theoretical recalculation [12] and several phenomenological analyses [9,13]. Third, the running strange quark mass which appears in the calculation of the matrix elements of the $(S-P)(S+P)$ penguin operator, the $SU(3)$ breaking effect in the involved η' decay constants and the normalization of the $B \to \eta^{(')}$ matrix element involved raise the branching ratio substantially. Finally, nonfactorizable contributions, which are parametrized by the $N_c^{\rm eff}$, gives the final answer for the largeness of exclusive $B \to \eta' K$ [14,15]. It is very interesting to see the impacts of these subtleties mentioned above on the the exclusive charmless B_s decays to an η' or η [16]. That is the main purpose of this talk [17].

2 Theoretical Framework

We begin with a brief description of the theoretical framework. The relevant effective $\Delta B = 1$ weak Hamiltonian is

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\Big[V_{ub}V_{uq}^*(c_1O_1^u+c_2O_2^u)+V_{cb}V_{cq}^*(c_1O_1^c+c_2O_2^c)-V_{tb}V_{tq}^*\sum_{i=3}^{10}c_iO_i\Big], \quad (1)$$

where $q = d, s$, and O_{3-6} are QCD penguin operators and O_{7-10} are electroweak penguin operators. $C_i(\mu)$ are the Wilson coefficients, which have been evaluated to the next-to-leading order (NLO) [18,19]. One important feature of the NLO calculation is the renormalization-scheme and -scale dependence of the Wilson coefficients (for a review, see [20]). In order to ensure the μ and renormalization scheme independence for the physical amplitude, the matrix elements, which are evaluated under the factorization hypothesis, have to be computed in the same renormalization scheme and renormalized at the same scale as $c_i(\mu)$. However, as emphasized in [14], the matrix element $\langle O\rangle_{\rm fact}$ is scale independent under the factorization approach and hence it cannot be identified with $\langle O(\mu)\rangle$. Incorporating QCD and electroweak corrections to the four-quark operators, we can redefine $c_i(\mu)\langle O_i(\mu)\rangle = c_i^{\rm eff}\langle O_i\rangle_{\rm tree}$, so that $c_i^{\rm eff}$ are renormaliztion scheme and scale independent. Then the factorization approximation is applied to the hadronic matrix elements of the operator O at the tree level. The numerical values for $c_i^{\rm eff}$ are shown in the last column of Table I, where $\mu = m_b(m_b)$, $\Lambda^{(5)}_{\overline{\rm MS}} = 225$ MeV, $m_t = 170$ GeV and $k^2 = m_b^2/2$ are used [14].

In general, there are contributions from the nonfactorizable amplitudes. Because there is only one single form factor (or Lorentz scalar) involved in the decay amplitude of $B\,(D) \to PP,\ PV$ decays (P: pseudoscalar meson, V: vector meson), the effects of nonfactorization can be lumped into the effective parameters $a_i^{\rm eff}$ [21]:

$$a_{2i}^{\rm eff} = c_{2i}^{\rm eff} + c_{2i-1}^{\rm eff}\left(\frac{1}{N_c}+\chi_{2i}\right), \qquad a_{2i-1}^{\rm eff} = c_{2i-1}^{\rm eff} + c_{2i}^{\rm eff}\left(\frac{1}{N_c}+\chi_{2i-1}\right), \quad (2)$$

where $c_{2i,2i-1}^{\rm eff}$ are the Wilson coefficients of the 4-quark operators, and nonfactorizable contributions are characterized by the parameters χ_{2i} and χ_{2i-1}. We can parametrize the nonfactorizable contributions by defining an effective number of colors $N_c^{\rm eff}$, called $1/\xi$ in [22], as $1/N_c^{\rm eff} \equiv (1/N_c) + \chi$. Different factorization approach used in the literature can be classified by the effective number of colors $N_c^{\rm eff}$. The so-called "naive" factorization discards all the nonfactorizable contributions and takes $1/N_c^{\rm eff} = 1/N_c = 1/3$, whereas the

"large-N_c improved" factorization [23] drops out all the subleading $1/N_c$ terms and takes $1/N_c^{\rm eff} = 0$. In principle, $N_c^{\rm eff}$ can vary from channel to channel, as in the case of charm decay. However, in the energetic two-body B decays, $N_c^{\rm eff}$ is expected to be process insensitive as supported by data [24]. If $N_c^{\rm eff}$ is process independent, then we have a generalized factorization. In this paper, we will treat the nonfactorizable contributions with two different phenomenological ways : (i) the one with "homogenous" structure, which assumes that $(N_c^{\rm eff})_1 \approx (N_c^{\rm eff})_2 \approx \cdots \approx (N_c^{\rm eff})_{10}$, and (ii) the "heterogeneous" one, which considers the possibility of $N_c^{\rm eff}(V+A) \neq N_c^{\rm eff}(V-A)$. The consideration of the "homogenous" nonfactorizable contributions, which is commonly used in the literature, has its advantage of simplicity. However, as argued in [14], due to the different Dirac structure of the Fierz transformation, nonfactorizable effects in the matrix elements of $(V-A)(V+A)$ operators are *a priori* different from that of $(V-A)(V-A)$ operators, i.e. $\chi(V+A) \neq \chi(V-A)$. Since $1/N_c^{\rm eff} = 1/N_c + \chi$, theoretically it is expected that

$$\begin{aligned} N_c^{\rm eff}(V-A) &\equiv \left(N_c^{\rm eff}\right)_1 \approx \left(N_c^{\rm eff}\right)_2 \approx \left(N_c^{\rm eff}\right)_3 \approx \left(N_c^{\rm eff}\right)_4 \approx \left(N_c^{\rm eff}\right)_9 \approx \left(N_c^{\rm eff}\right)_{10}, \\ N_c^{\rm eff}(V+A) &\equiv \left(N_c^{\rm eff}\right)_5 \approx \left(N_c^{\rm eff}\right)_6 \approx \left(N_c^{\rm eff}\right)_7 \approx \left(N_c^{\rm eff}\right)_8, \end{aligned} \tag{3}$$

To illustrate the effect of the nonfactorizable contribution, we extrapolate $N_c(V-A) \approx 2$ from $B \to D\pi(\rho)$ [25] to charmless decays.

Table 1: Numerical values of effective coefficients a_i at $N_c^{\rm eff} = 2, 3, 5, \infty$, where $N_c^{\rm eff} = \infty$ corresponds to $a_i^{\rm eff} = c_i^{\rm eff}$. The entries for a_3,...,a_{10} have to be multiplied with 10^{-4}.

	$N_c^{\rm eff} = 2$	$N_c^{\rm eff} = 3$	$N_c^{\rm eff} = 5$	$N_c^{\rm eff} = \infty$
a_1	0.986	1.04	1.08	1.15
a_2	0.25	0.058	-0.095	-0.325
a_3	$-13.9 - 22.6i$	61	$120 + 18i$	$211 + 45.3i$
a_4	$-344 - 113i$	$-380 - 120i$	$-410 - 127i$	$-450 - 136i$
a_5	$-146 - 22.6i$	-52.7	$22 + 18i$	$134 + 45.3i$
a_6	$-493 - 113i$	$-515 - 121i$	$-530 - 127i$	$-560 - 136i$
a_7	$0.04 - 2.73i$	$-0.7 - 2.73i$	$-1.24 - 2.73i$	$-2.04 - 2.73i$
a_8	$2.98 - 1.37i$	$3.32 - 0.9i$	$3.59 - 0.55i$	4
a_9	$-87.9 - 2.73i$	$-91.1 - 2.73i$	$-93.7 - 2.73i$	$-97.6 - 2.73i$
a_{10}	$-29.3 - 1.37i$	$-13 - 0.91i$	$-0.04 - 0.55i$	19.48

The $N_c^{\rm eff}$-dependence of the effective parameters a_i's are shown in Table I, from which we see that a_1, a_4, a_6 and a_9 are $N_c^{\rm eff}$-stable, and the remaining ones are $N_c^{\rm eff}$-sensitive. We would like to remark that while a_3 and a_5 are both $N_c^{\rm eff}$-sensitive, the combination of $(a_3 - a_5)$ is rather stable under the

variation of the $N_c^{\rm eff}$ within the "homogeneous" picture and is still sensitive to the factorization approach taken in the "heterogeneous" scheme. This is the main difference between the "homogeneous" and "heterogeneous" approaches. While a_7, a_8 can be neglected, a_3, a_5 and a_{10} have some effects on the relevant processes depending on the choice of $N_c^{\rm eff}$.

3 Phenomenology

Table 2: Average branching ratios (in units of 10^{-6}) for charmless B_s decays to η' and η. Predictions are for $k^2 = m_b^2/2$, $\eta = 0.34$, $\rho = 0.16$. I denotes the "homogeneous" nonfactorizable contributions i.e. $N_c^{\rm eff}(V-A) = N_c^{\rm eff}(V+A)$ and (a,b,c,d) represent the cases for $N_c^{\rm eff}$=(∞,5,3,2). II denotes the "heterogeneous" nonfactorizable contributions, i.e. $N_c^{\rm eff}(V-A) \neq N_c^{\rm eff}(V+A)$ and (a',b',c') represent the cases for $N_c^{\rm eff}(V+A)$=(3,5,∞), where we have fixed $N_c^{\rm eff}(V-A)$=2 (see the text)

Decay	I_a	I_b	I_c	I_d	$II_{a'}$	$II_{b'}$	$II_{c'}$
$\bar{B}_s \to \pi\eta'$	0.25	0.17	0.13	0.11	0.11	0.11	0.10
$\bar{B}_s \to \pi\eta$	0.16	0.11	0.08	0.07	0.07	0.068	0.067
$\bar{B}_s \to \rho\eta'$	0.70	0.47	0.36	0.30	0.30	0.30	0.31
$\bar{B}_s \to \rho\eta$	0.45	0.30	0.24	0.19	0.19	0.19	0.20
$\bar{B}_s \to \omega\eta'$	6.9	0.9	0.012	2.14	0.48	0.03	0.83
$\bar{B}_s \to \omega\eta$	4.45	0.63	0.008	1.39	0.31	0.02	0.54
$\bar{B}_s \to \eta' K^0$	1.25	1.07	1.01	1.00	1.27	1.51	1.90
$\bar{B}_s \to \eta K^0$	1.35	0.81	0.68	0.76	0.75	0.74	0.72
$\bar{B}_s \to \eta' K^{*0}$	0.49	0.35	0.32	0.26	0.49	0.60	0.80
$\bar{B}_s \to \eta K^{*0}$	0.45	0.05	0.02	0.24	0.24	0.24	0.25
$\bar{B}_s \to \eta\eta'$	47.4	41.8	38.3	34.4	39.5	44.1	51.5
$\bar{B}_s \to \eta'\eta'$	26.6	24.9	23.8	22.4	33.8	43.9	62.2
$\bar{B}_s \to \eta\eta$	20.3	17.1	15.1	12.8	11.6	10.7	9.1
$\bar{B}_s \to \phi\eta'$	0.44	0.59	2.29	6.20	4.41	3.11	1.66
$\bar{B}_s \to \phi\eta$	0.04	0.91	2.29	4.92	2.28	0.92	0.10

With the following input parameters, we obtain the branching ratios shown in Table 2.

- For the running quark masses, we use [26]

$$m_u(m_b) = 3.2\,\mathrm{MeV}, \quad m_d(m_b) = 6.4\,\mathrm{MeV}, \qquad m_s(m_b) = 105\,\mathrm{MeV},$$
$$m_c(m_b) = 0.95\,\mathrm{GeV}, \quad m_b(m_b) = 4.34\,\mathrm{GeV}, \tag{4}$$

- The Wolfenstein parameters with $A = 0.81$, $\lambda = 0.22$, $\rho = 0.16$, and $\eta = 0.34$ are used in this work.

- For values of the decay constants, we use $f_\pi = 132$ MeV, $f_K = 160$ MeV, $f_\rho = 210$ MeV, $f_{K^*} = 221$ MeV, $f_\omega = 195$ MeV and $f_\phi = 237$ MeV. For

the matrix element, we use the relativistic quark model's results [27] with a proper normalization.

From this studies, we learned that

- Similar to their $B_{u,d}$ corresponding decay modes, $B_s \to \eta(')\eta(')$ have the largest branching ratios ($O(10^{-5})$ and thus are the interesting modes to be observed in the near future.

- Since the internal W-emission is CKM-suppressed and the QCD penguins are canceled out in these decay modes, $\bar{B}_s \to \pi(\rho)\eta^{(')}$ are dominated by the EW penguin diagram. The dominant EW penguin contribution proportional to a_9 is $N_c^{\rm eff}$-stable. Thus, by measuring these branching ratios, we can determine the effective coefficient a_9.

- It is found that for processes depending on the $N_c^{\rm eff}$-stable a_i's such as $\bar{B}_s \to (\pi, \rho)\eta^{(')}$, the branching ratios are not sensitive to the factorization approach we used. While for the processes depending on the $N_c^{\rm eff}$-sensitive a_i's such as the $\bar{B}_s \to \omega\eta^{(')}$, the predicted branching ratios have a wide range depending on the choice of the factorization approach. It means that even within the standard model, there are large uncertainties for these $N_c^{\rm eff}$-sensitive processes.

- For the mechanism $(c\bar{c}) \to \eta'$,in general, it has smaller effects due to a possible CKM-suppression and the suppression in the decay constants except for the $\bar{B}_s \to \phi\eta$ under the "large-N_c improved" factorization approach, where the internal W diagram is CKM-suppressed and the penguin contributions are compensated.

4 *Summary and Discussions*

We have studied charmless exclusive nonleptonic B_s meson decay into an η or η' within the generalized factorization approach. Nonfactorizable contributions are parametrized in terms of the effective number of colors $N_c^{\rm eff}$ and predictions using different factorization approaches are shown with the $N_c^{\rm eff}$ dependence.

In our work, we, following the standard approach, have neglected the W-exchange and the space-like penguin contributions. Another major source of uncertainties comes from the form factors we used, which are larger than the BSW model's calculations. For simple processes such as $B_s \to \pi(\rho,\omega)\eta(')$, they only scale with a factor, while for the complicated processes like $B_s \to K^0\eta(')$ the different contributions (tree, QCD penguin, EW penguin) will have different weights. Although the Wolfenstein parameter ρ ranges from the negative

region to the positive one, we have "fixed" it to some representative values. The interference pattern between the internal W diagram and the penguin contributions will change when we take a different sign of ρ.

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